NONLINEAR ANALYSIS OF SOLAR RADIO EVENTS: A PRELIMINARY APPROACH

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Abstract

We analysed a set of time series related to different types of solar radio events (type I, type IV and spikes) in order to determine the nature of the underlying generating process through the methods of nonlinear dynamics. The Grassberger-Procaccia method was used to evaluate the correlation dimension of a possible attractor in subsets which fulfilled the stationarity condition. The majority of samples do not show a low dimensionality, suggesting stochasticity or a high dimensional system; only two overlapping subsets of one type IV event with spikes exhibit a finite dimension ($D \sim 3.5$ and $D \sim 3.7$). The limited datasets do not allow to draw any definite conclusion, but the varied results indicate that a critical analysis on the physical character of solar radio events is needed to give a consistent interpretation.

1 Introduction

Solar radio events are the signature of different kinds of plasma processes occuring in the solar atmosphere. The underlying physics is quite complex and generally the observable, i.e., the radio flux density, is not periodic in the time domain. A fundamental goal of the analysis of the relevant time series consists in determining if the originating process has a stochastic or a deterministic character, as such information must be considered in the construction of models.

The solution of simple, nonlinear equations can exhibit a very complicated behaviour - chaotic behaviour, which is nevertheless deterministic. Therefore to try to derive a consistent model of the original system in terms of nonlinear equations, we made use of the methods of nonlinear dynamics theory, which is to be applied when the linear mode theory does not attain satisfactory results.

A variety of parameters can be used to quantify nonlinear dynamical systems, such as dimensions, Kolmogorov entropy and Lyapunov exponents [Eckmann and Ruelle, 1985; Grassberger et al., 1991]. They give a characterization of the attractor, i.e., the limit set

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of trajectories of the dynamical system in phase space after all transient phenomena have died out. The Kolmogorov entropy measures the information flux rate of the system. The Lyapunov exponents are associated with the complexity of the dynamics of the trajectories on the attractor: chaotic systems are highly sensitive to initial conditions, this phenomenon is quantified by the Lyapunov exponents. The dimensions of attractors measure the complexity of the attractor's geometry in phase space. If the system is chaotic the related attractor is extremely complex. Being fractal it has non-integer dimension, therefore called *strange attractor*.

As the dimension of an attractor represents a measurement of the complexity of the attractor's geometry it indicates the number of degrees of freedom of the system. The dimension for stochastic systems (noise) is infinite, whereas chaotic systems exhibit finite dimensions. Therefore, determining the dimension of an attractor allows to discriminate if an irregular process is stochastic or deterministic (even if chaotic).

In our analysis we used a set of 1-D solar radio data (flux density time series) of different types and followed the prescriptions given by Isliker and Benz [1994b]. The preliminary results we obtained are consistent with those of the cited authors and emphasize the complexity of the phenomena under examination.

The analysis methodology is sketched in Section 2. In Section 3 we describe the datasets and show the results. Related considerations on the generation of solar radio events are presented in Section 4. We comment the results in Section 5, stressing the relevant points for a further analysis with emphasis on the intrinsic nature of the originating physical systems.

2 Analysis methodology

In this preliminary analysis we restricted our attention to the derivation of the correlation dimension, which characterizes an attractor of the system. The correlation dimension $D^{(2)}$ is one out of many definitions of fractal dimensions. It is based on the idea that the number of points which are closer to each other than a distance r scales with the dimension of the attractor as

$$C(r) \propto r^{D^{(2)}}, \quad r \to 0 \tag{1}$$

After the Grassberger-Procaccia method [Grassberger and Procaccia, 1983a,b] $D^{(2)}$ is estimated by evaluating the correlation integral $C(r)$:

$$C(r) = \frac{1}{N^2} \times \text{ number of pairs } (\boldsymbol{\xi}_i, \boldsymbol{\xi}_j) \text{ with } |\boldsymbol{\xi}_i - \boldsymbol{\xi}_j| \leq r \quad (i \neq j)$$

with N the overall number of data points. For calculating $D^{(2)}$ only spatial correlations, i.e., correlations in phase space have to be considered; pairs of points which are only close to each other because they are temporally close have to be excluded. The correlation integral becomes then:

$$C(r) := \lim_{N\to\infty} \frac{2}{(N-W)(N-W+1)} \sum_{i+W<j}^{N} \theta(r - |\boldsymbol{\xi}_i - \boldsymbol{\xi}_j|) \tag{2}$$

$\theta(x)$ is the Heaviside function, defined as

$$\theta(x) := \begin{cases} 0 & \text{for} \quad x \leq 0 \\ 1 & \text{for} \quad x > 0 \end{cases}$$

The parameter W which excludes temporal correlations should not be less then the autocorrelation time [Theiler, 1986]. For small distances r the correlation dimension is obtained from the relation

$$D^{(2)} = \lim_{r\to 0} \frac{\ln C(r)}{\ln r} \tag{3}$$

Often in astronomy one is dealing with one-dimensional time series $\{x(t_i)\}$. To determine the dimension of an attractor the m-dimensional phase space of the system has to be reconstructed. This is usually done by time delay techniques. Each measured value $x(t_i)$ of the time series is replaced by a vector

$$\boldsymbol{\xi}_i^{(m)} = \{x(t_i)\ x(t_i+\tau)\ \ldots\ x(t_i+(m-1)\tau)\} \tag{4}$$

These vectors build up the reconstructed phase space of dimension m. τ is the time delay, it is a multiple of the time resolution $\triangle t = t_{i+1} - t_i$. If $m \geq 2D+1$ then the embedding of the attractor in the reconstructed phase space is ensured [Takens, 1981]. For real data (finite length of the time series, presence of noise) the time delay τ has to be chosen with care. If τ is chosen too large the components of $\boldsymbol{\xi}_i^{(m)}$ become independent of each other (no causal connection). Otherwise too small τ leads to coordinates which are too strongly correlated, the $\boldsymbol{\xi}_i^{(m)}$ are not sufficiently spread in phase space.

In practice one computes the correlation integral $C(r)$ by Equation (2) for increasing embedding dimensions m as shown in Figure 1. Then the dimension $D^{(2)}$ of the attractor is calculated from the slopes of the curves

$$\ln C(r) = D^{(2)} \ln r \tag{5}$$

If $D^{(2)}(m)$ becomes constant with increasing embedding dimension m for relatively small m there exists an attractor of the time series (Figure 2).

For real data the power law scaling for $D^{(2)}$ is only fulfilled in a restricted range of r. For small distances r noise is dominating, at large r the correlation integral diverges from the power law scaling due to the finite length of the data set. A reliable way to find

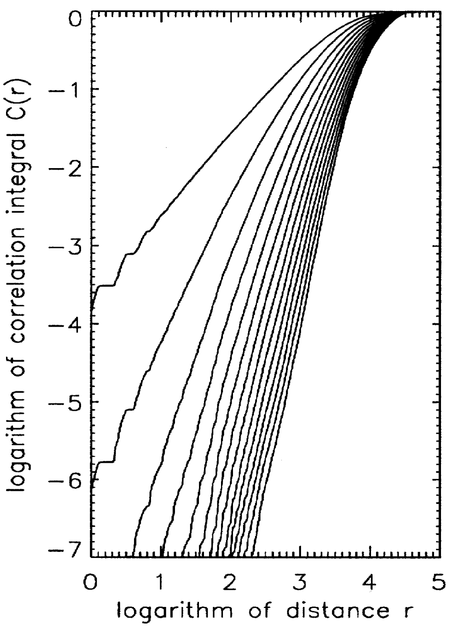

Figure 1: Logarithm of the correlation integral $C(r)$ versus logarithm of distance r for embedding dimension $m = 1$ up to $m = 14$ for a converging type IV with spikes (10/02/84, pts 3950:5100).

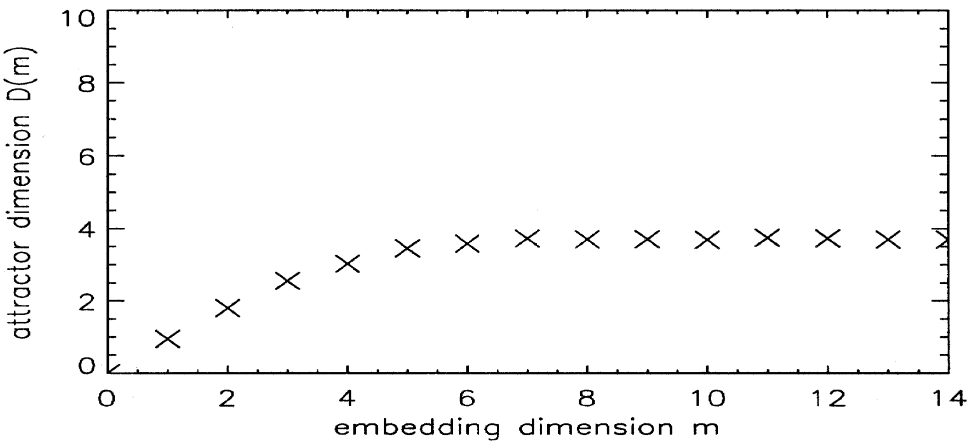

Figure 2: Correlation dimension $D^{(2)}(m)$ versus embedding dimension m. The convergence of $D^{(2)}(m)$ with increasing m exhibits low-dimensional chaos: $D^{(2)} \sim 3.7$.

out the scaling region is to consider the graphs of local slopes $d\,[\ln C(r)]\,/d\,[\ln r]$ versus $\ln r$ as shown in Figure 3a for a sample event (type IV with spikes). In many cases the plateau region is better marked and of larger extension in the modified graph of local slopes $d\,[\ln C(r)]\,/d\,[\ln r]$ versus $\ln C$ (Figure 3b). There are varied phenomena affecting the quality of the plateau region: finite length of the time series, noise, inhomogeneity and lacunarity of the attractor (meaning very sparse populated attractor regions). These effects lead to small, skew [Smith, 1988] and deformed plateau regions. In the worst case no plateau is reliably detectable.

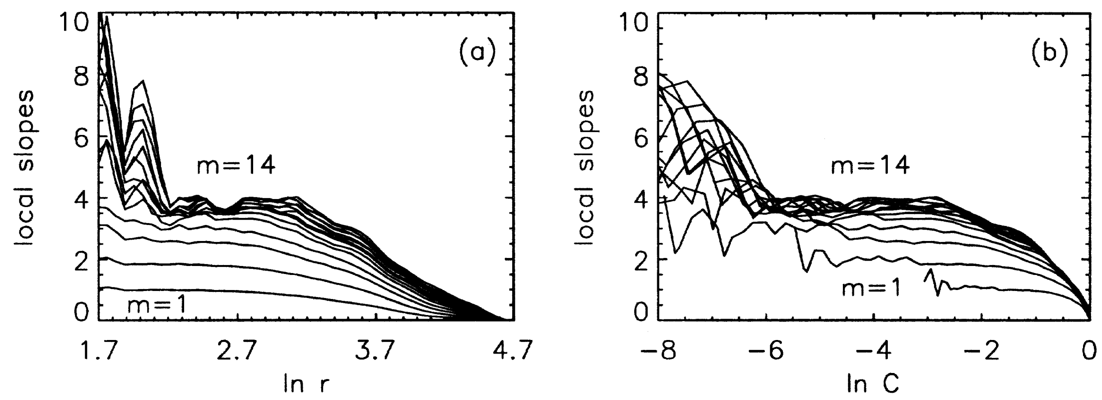

Figure 3: Graph of local slopes: (a) $d[\ln C(r)]/d[\ln r]$ versus $\ln r$, (b) $d[\ln C(r)]/d[\ln r]$ versus $\ln C$. The plateau indicates the linear scaling range of $\ln C(r)$ for each embedding dimension m.

In particular many conditions exist about the minimum length of the time series required to determine the dimension of the attractor. Eckmann and Ruelle [1992] derived a condition for the minimal data length, valid for noisefree data. The required number of points N of the data set is given by

$$D < 2\log_{10} N \tag{6}$$

Additionally restricting conditions are that the time series is sampled with about 10–30 points per significant structure (characteristic time scale of the process) and contains a sufficient number (> 50) of these large scale structures [Brandstater and Swinney, 1987; Kurths et al., 1991].

3 Datasets and results

We analysed datasets of type I storms (6 samples, 2 at 2 frequencies), type IV bursts (3 samples, 1 at 3 frequencies), type IV bursts with spikes (2 samples) and isolated spikes (1 sample at 2 frequencies). The datasets are high time resolution ($10-50$ Hz), single frequency recordings from the multichannel radio polarimeter of the Trieste Observatory operating in the metric range ($200-800$ MHz). For each event it was selected for analysis the time series representing the predominant polarization sense during its evolution, i.e., the left or right circular polarization channel. A sample time series of a type IV with spikes is shown in Figure 4.

As the attractor of a system is the limit set of the trajectories after all transient phenomena have died out only events which have reached stationary phases are suited for further analysis. To find out stationary subsections we have used a test for stationarity proposed by Isliker and Kurths [1993]. This test gives a necessary condition for stationarity. The invariant measure of the entire section and the first half of it are estimated and compared by a χ^2-test (confidence level 95%). The application of such test resulted in the identification of 23 stationary subsets for type I bursts, 8 for type IV bursts, 5 for type IV

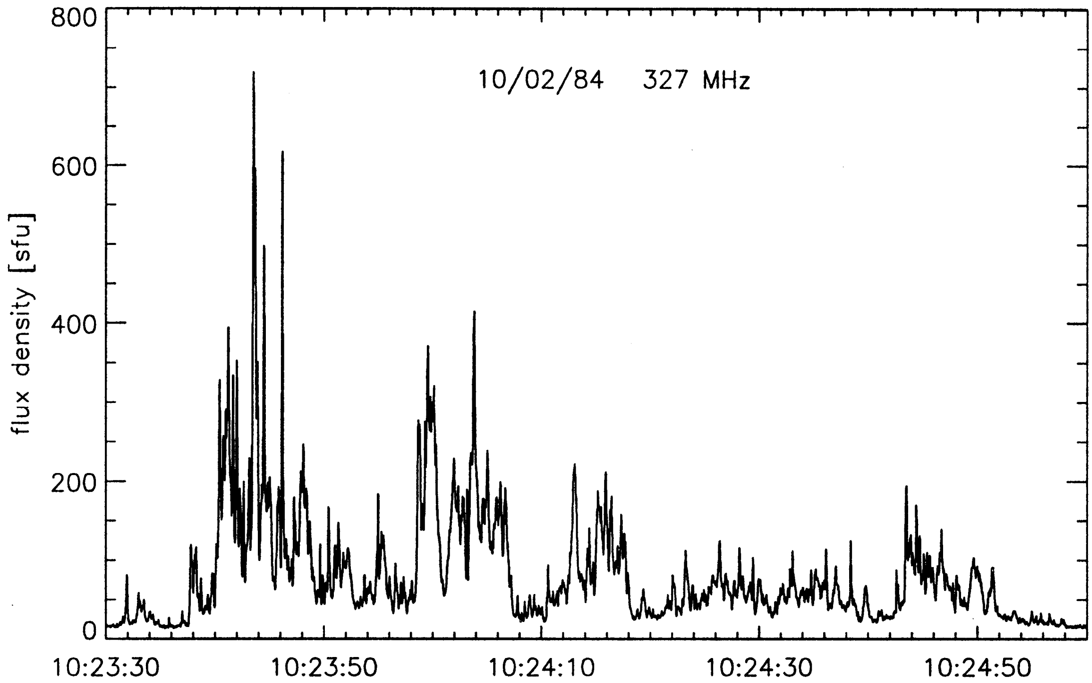

Figure 4: Time series of a type IV event with spikes.

plus spikes, and 2 for isolated spikes. Therefore a total of 38 subsets, sometimes partially overlapping, was searched for an attractor's dimension.

Applying the above mentioned conditions about the minimum length of the time series to the investigated data sets we must not expect to detect dimensions $D > 5$ and also results below this value must not be expected to be of high accuracy.

No convergence was found for the majority of events but two overlapping subsets of a type IV burst with spikes, which are characterized by a dimension of the order 3.5 and 3.7 indicating the possible existence of an attractor. The results are summarized in Table 1. Type I storms and type IV bursts exhibit no low dimensionality. This behavior may be suggestive of a high dimensionality (too high to detect with our tools) or of a stochasticity of the process. In some subsets of one type I storm only very small plateau regions could be detected. The reason might be that the related data sets are too noisy for such kind of analysis (and in fact this noise storm is not very intense at the frequency of 408 MHz). Some subsets of a type IV burst observed at two different frequencies show a deformed plateau indicating that the related attractor is of great inhomogeneity and/or lacunarity. As mentioned above only two subsets of one type IV with spikes have a defined dimension, the others showing divergence as well as the two subsets with spikes. To comment on such results in the following section we briefly speculate on the physical nature of solar radio events.

Date	Start	Resol. [ms]	Freq. [MHz]	Event type	Stationary sections [pts]	Dimension
01/02/84	09:53:00	20	237	I	2200:5000	div.
					3000:4550	div.
26/04/84	13:25:30	20	237	I	1250:2700	div.
					1700:2600	div.
26/04/84	16:00:00	20	237	I	7000:10000	div.
11/05/91	11:00:00	20	237	I	500:12000	div.
					2000:4000	div.
					4000:7500	div.
					7500:11000	div.
09/06/94	14:40:00	100	327	I	120:6000	div.
					120:2400	div.
					2400:4200	div.
					4200:6000	div.
					1800:4800	div.
			408		3000:6000	div. (sm. pl.)
	14:50:00		327		120:3000	div.
					3000:6000	div.
					800:2900	div.
			408		120:5990	div. (sm. pl.)
					120:3600	div. (sm. pl.)
					3600:5990	div. (sm. pl.)
					2800:4500	div.
					2900:3500	div.
10/02/84	14:44:00	20	237	IV	4650:5900	div.
					4000:5000	div. (def. pl.)
			327		4000:5300	div. (def. pl.)
			408		4700:5300	div.
					4000:5300	div. (def. pl.)
					4000:5900	div. (def. pl.)
09/03/89	13:05:20	20	408	IV	0:600	div.
	13:05:32				0:900	div.
10/02/84	10:23:00	20	327	IV+s	1900:3500	div.
					1900:3900	div.
					3500:5600	conv.: $D \sim 3.5\,(m \geq 6)$
					3950:5100	conv.: $D \sim 3.7\,(m \geq 7)$
10/02/84	10:57:00	20	327	IV+s	1200:1800	div.
17/03/84	10:30:50	20	327	s	800:1550	div.
			408		800:1550	div.

Table 1: Results of the dimension derivation for the analysed datasets. Stationary subsets are identified through point sequential indexing. Event type: I, IV, s(pikes). Key to abbreviations: div., divergence, meaning that the correlation dimension is increasing with embedding dimension; conv. convergence, indicating the possible existence of a low dimensional attractor with dimension D; no pl., no plateau in the curves of local slopes could be found; *def. pl.*, deformed plateau, a phenomenon of inhomogeneity and lacunarity of the attractor; *sm. pl.*, small plateau region.

4 The nature of solar radio events

The generation of solar radio events through the plasma radiation mechanism occurs via the triggering and the saturation of a plasma instability which originates radio waves through a nonlinear process in a plasma region whose spatial and time scales characterize the event's type (Figure 5).

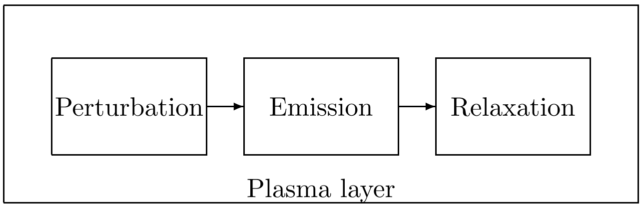

Figure 5: States of a plasma layer originating radio emission.

4.1 Isolated radio events

The observation of an *isolated radio event* means that only one radio source is operating during the observation time. The region of plasma which is effected by the radio emission process can have different spatial scales. An *extended* region usually generates emission on a *larger* time scale, i.e., the *background* or *continuum component*. A *localized* region instead produces radiation on a *smaller* time scale, i.e., the *transient* or *burst component*.

Therefore in the case of isolated events, we can assume that each type of emission (continuum or burst) is to be related to a unique radio source, i.e., plasma region with its characteristic spatial scale which is radioemitting alone and independently.

Continuum radiation is seldom observed as an isolated feature, but it can occur sometimes, e.g., as type IV continuum which exhibits rise, maximum and decline phases. More common is the observation of isolated bursts such as type III bursts.

4.2 Complex radio events

In solar radio astronomy one usually observes *complex radio events*, which exhibit a continuum component and a burst component superimposed to the continuum one. In such a case the observed radio source is complex and it is composed by an extended region originating the continuum underlying many localized sources producing the bursts.

As the observed features of continuum and bursts in a complex event maintain the same character of similar isolated events, we can assume that the sources of each component undergo the same kind of plasma process and radiate, as a first approximation, independently but at the same time. In particular, the continuum source is emitting on a time scale much larger than that of the bursts' sources, which, in turn, are usually radiating not all at the same time but at subsequent times with an eventual partial overlapping

(Figure 6). In reality at least a weak coupling should exist among the different component sources which we can assume it is realized through the local magnetic field and its topological changes.

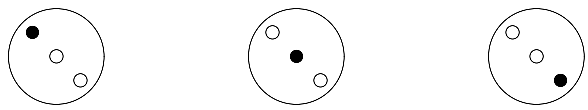

Figure 6: A complex radio source observed at different subsequent times: the larger continuum source is continuously radiating whereas the localized burst sources are radiating in turn.

Typical examples of complex events are type I storms (noise storms), type III storms and type IV bursts intended as continuum with fine structures.

4.3 Characteristics of observed time series

The general form of a time series representing a complex solar radio event can be written as the sum of different components, each of them with a characteristic time scale and related to a specific radio source (spatial scale):

$$S(t;f) = \sum_{\tau_f} \sum_i S_{\tau_f\, i}(t) \tag{7}$$

where τ_f is the *characteristic time scale*, which depends on the emission frequency f and on each component's type, and i is the *number of components*, which determines the typical time evolution of the complex event.

For example, in case of a time series observed in the decimetric band, the characteristic time scale may assume the following order of magnitudes for the different components: $\tau \sim 1\ ms \rightarrow$ *spike*, $\tau \sim 0.1\ s \rightarrow$ *type I burst*, $\tau \sim 1\ s \rightarrow$ *type III burst*, $\tau \sim 10\ min \rightarrow$ *type I or type IV continuum*.

According to Equation (7), a model time series representing a complex radio event can be written as

$$S(t;f) = S_{\tau_{cf}}(t) + \sum_i S_{\tau_{bf}\, i}(t) \tag{8}$$

where the continuum component can be represented by a polynomial functional as

$$S_{\tau_{cf}}(t) = \sum_{k=0}^{N} a_{k\,f} t^k \tag{9}$$

and the burst component by a sum of time-shifted gaussians as

$$S_{\tau_{bf}\, i}(t) = \left(\frac{1}{\sqrt{2\pi}\Delta t_f} \right) exp\left[-\frac{1}{2\Delta t_f^2}(t-t_i)^2 \right] \tag{10}$$

where $\tau_{cf} \gg \tau_{bf} = \Delta t_f$. An appropriate choice of time scales and mathematical forms of the components allows in principle to simulate any observed time series.

5 Discussion and conclusion

No final conclusion can be drawn from this preliminary analysis of a limited dataset, nevertheless it raises a fundamental point to be considered in any further development. In particular, according to our opinion both the results by previous authors [Isliker and Benz, 1994b] and our results depend critically not only on the interpretation of the analysis, which could lead to non-consistent conclusions, but especially on the a priori analysis of the nature of the data themselves and hence in their selection.

In case of the time series we have analysed, the conventional techniques allow to determine the existence of an attractor with a maximum dimension of about 5, which still represents a moderate level of complexity in a chaotic determinism. But if the system has a higher dimensionality, because it is too complex, or we are looking at a signature which is the output of more systems operating at the same time, we cannot hope to be able to discriminate between stochasticity or determinism. A proper selection of the events to be analysed could help in this sense, if we try to identify samples with defined features possibly produced by the same source and, to a certain extent, the analysis of the polarization is indicative of the source unity for some type of events. For example, the negative results for type I storms could be due to the fact that all the time series but those with intermediate polarization are the signatures from multiple sources on the disk and therefore, even if a determinism exists, it is masked by the combined action of more systems. The convergence was found only in two subsets where a unique component with chaotic nature was isolated.

A critical review of solar radio events, i.e., their generation mechanism and observational properties, is the only way to understand which of them is suitable to such kind of analysis; the required fulfilment of conditions about stationarity, intermittency, etc. might not hold for all classes of radio events. In fact, most types are complex in the sense that a complex source is active which can be hardly considered as a unique physical system with a simple attractor but is instead a set of systems coupled together, more probably with a stochastic character or a high dimensionality, which is indistinguishable by the former. As a next step, we intend to develop such a critical task of identifying the most suitable events also by means of numerical simulations of radio time series to investigate the actual potentialities of the quantifiers.

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